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1. Písemná práce – oddělení B – třída 3. A4 – 16. 11. 2015

5 b. 1. Vypočítejte:

1 b. a) $\cos 45° = \frac{\sqrt{2}}{2}$

1 b. b) $\cos^2 45° = (\cos 45°)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

1 b. c) $\cos (45°)^2 = \cos(45° \cdot 45°) = \cos 2025° = \cos(225° + 5 \cdot 360°) = \cos 225° = -\frac{\sqrt{2}}{2}$

1 b. d) $\cos^2 (45°)^2 = \cos(45°)^2 \cdot \cos(45°)^2 = \left(-\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

1 b. e) $4 \cdot \cos^2 (45°)^2 = 2$

11 b. 2. Řešte v R rovnice:

1 b. a) $\cos x = 0{,}5$ $K_R = \{\frac{\pi}{3} + 2k\pi; \frac{5}{3}\pi + 2k\pi\}$

1 b. b) $\cos x = -0{,}5$ $K_R = \{\frac{2}{3}\pi + 2k\pi; \frac{4}{3}\pi + 2k\pi\}$

2 b. c) $\cos (2x + \frac{\pi}{2}) = 0{,}5$

$2x + \frac{\pi}{2} = \frac{\pi}{3} + 2k\pi \lor 2x + \frac{\pi}{2} = \frac{5}{3}\pi + 2k\pi$ 1 b.

$2x = -\frac{\pi}{6} + 2k\pi \lor 2x = \frac{7}{6}\pi + 2k\pi$

$x = -\frac{\pi}{12} + k\pi \lor x = \frac{7}{12}\pi + k\pi$

$K_R = \{-\frac{\pi}{12} + k\pi; \frac{7}{12}\pi + k\pi\}$ 1 b.

2 b. d) $\sin (2x - \frac{\pi}{4}) = -0{,}5$

$2x - \frac{\pi}{4} = \frac{7}{6}\pi + 2k\pi \lor 2x - \frac{\pi}{4} = \frac{11}{6}\pi + 2k\pi$ 1 b.

$2x = \frac{17}{12}\pi + 2k\pi \lor 2x = \frac{25}{12}\pi + 2k\pi$

$x = \frac{17}{24}\pi + k\pi \lor x = \frac{25}{24}\pi + k\pi$

$K_R = \{\frac{17}{24}\pi + k\pi; \frac{25}{24}\pi + k\pi\}$ 1 b.

2 b. e) $\operatorname{tg} (x - \pi) = \frac{\sqrt{3}}{3}$

$x - \pi = \frac{\pi}{6} + k\pi$ 1 b.

$x = \frac{7}{6}\pi + k\pi$

$K_R = \{\frac{7}{6}\pi + k\pi\}$ 1 b.

3 b. e) $2 \cos^2 x - 5 \sin x + 1 = 0$ $\cos^2 x = 1 - \sin^2 x$

$2(1 - \sin^2 x) - 5\sin x + 1 = 0$

$2 - 2\sin^2 x - 5\sin x + 1 = 0$

$2\sin^2 x + 5\sin x - 3 = 0$ 1 b.

$D = 25 + 24 = 49;\ \sqrt{D} = 7$

$(\sin x)_{1,2} = \frac{-5 \pm 7}{4} = \begin{cases} \frac{1}{2} \\ -3 \end{cases}$ 1 b.

$\sin x = \frac{1}{2} \lor \sin x = -3$

$K_R = \{\frac{\pi}{6} + 2k\pi; \frac{5}{6}\pi + 2k\pi\}$ 1 b.

5 b. 3. Narýsujte grafy funkcí v intervalu <0; 2π> (grafy rozlište jasně barvou či druhem čáry):

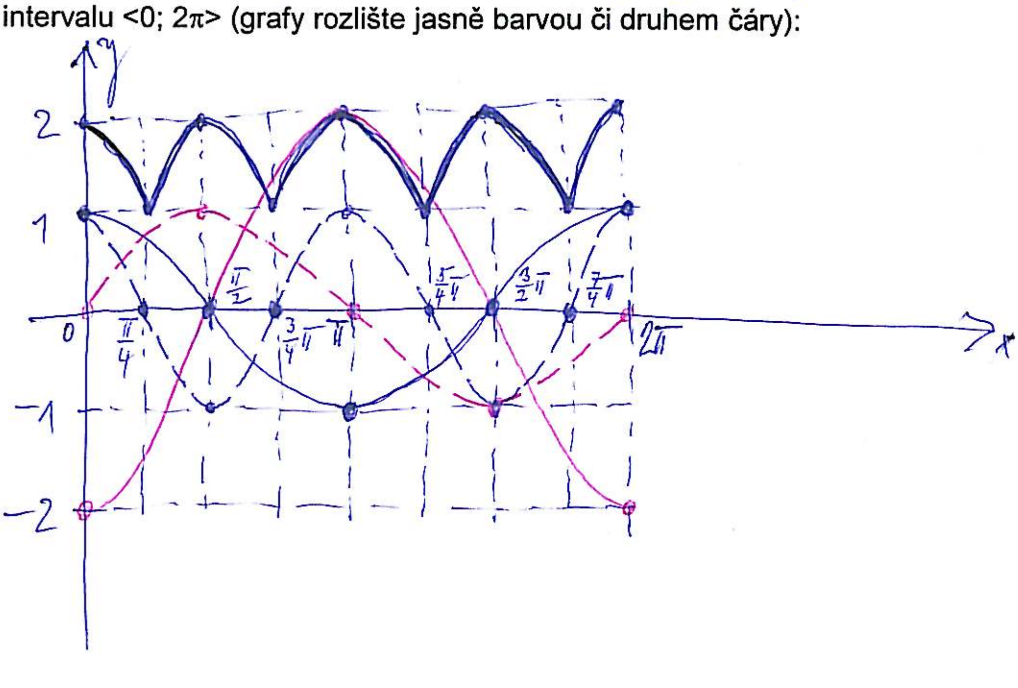
1 b. → a) $y = \cos x$

1 b. → b) $y = -2\cos x$

1 b. → c) $y = \cos(x - \frac{\pi}{2})$

1 b. → d) $y = \cos 2x$

1 b. → e) $y = |\cos 2x| + 1$



(3)

(6b.)

4. Pomocí vhodného vzorce zjednodušte daný výraz:

2b. a) $\sin(x-\pi) = \sin x \cos\pi - \cos x \sin\pi = \sin x \cdot (-1) - \cos x \cdot 0 = \underline{\underline{-\sin x}}$

2b. b) $\cos(x+\pi) = \cos x \cos\pi - \sin x \sin\pi = \cos x \cdot (-1) - \sin x \cdot 0 = \underline{\underline{-\cos x}}$

1b. ... správně použitý vzorec
1b. ... -„- vyjádření

2b. c) $\sin(x+\pi) + \sin(x-\pi) = \sin x \cos\pi + \cos x \sin\pi + \sin x \cos\pi - \cos x \sin\pi =$

$= 2\sin x \cos\pi = 2\sin x \cdot (-1) = \underline{\underline{-2\sin x}}$

nebo

$\sin(x+\pi) + \sin(x-\pi) = 2\sin\frac{(x+\pi)+(x-\pi)}{2}\cos\frac{(x+\pi)-(x-\pi)}{2} = -2\sin x$

(6b.)

5. Zjednodušte daný výraz, určete definiční obor výrazu:

2b. a) $\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 = 1^2 = \underline{1}$ $\quad D = R$

1b. ... správné zjednodušení
1b. ... -„- určení D

2b. b) $\sin^2 x + \sin^2 x \cdot \text{cotg}^2 x = \sin^2 x + \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x} = \sin^2 x + \cos^2 x = \underline{\underline{1}}$

$D = R - \{k\pi\}$

2b. f) $\frac{\cos^4 x - \sin^4 x}{\cos 2x} = \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\cos^2 x - \sin^2 x} = \cos^2 x + \sin^2 x = \underline{\underline{1}}$

$\cos 2x \neq 0$
$2x \neq \frac{\pi}{2} + k\pi$
$x \neq \frac{\pi}{4} + k\cdot\frac{\pi}{2}$

$D = R - \{\frac{\pi}{4} + k\cdot\frac{\pi}{2}\}$

(2b.)

6. Dokažte, že pro všechna x, y ∈ R platí: sin (x + y) + sin (x – y) = 2 · sin x · cos y.

$\sin(x+y) + \sin(x-y) = \sin x\cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y =$
$= \underline{2\sin x \cos y}$

nebo

$\sin(x+y) + \sin(x-y) = 2\sin\frac{(x+y)+(x-y)}{2}\cdot\cos\frac{(x+y)-(x-y)}{2} =$

$= \underline{\underline{2\sin x \cdot \cos y}}$

(4b.)

7. Vypočítejte cos x, tg x a cotg x, znáte-li: $\sin x = \frac{3}{5} \wedge x \in (\frac{\pi}{2}; \pi)$.

$x \in (\frac{\pi}{2};\pi) \Rightarrow \cos x < 0 \wedge \text{tg}\, x < 0 \wedge \text{cotg}\, x < 0$

$\cos^2 x + \sin^2 x = 1 \Rightarrow |\cos x| = \sqrt{1-\sin^2 x} = \sqrt{1-(\frac{3}{5})^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5} \Rightarrow$

$\Rightarrow \underline{\underline{\cos x = -\frac{4}{5}}}$ 2b.

$\text{tg}\, x = \frac{\sin x}{\cos x} = \frac{\frac{3}{5}}{-\frac{4}{5}} = \underline{\underline{-\frac{3}{4}}}$ $\qquad \underline{\underline{\text{cotg}\, x = -\frac{4}{3}}}$

1b. $\qquad$ 1b.