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1. Písemná práce – oddělení A – třída 3. A4 – 16. 11. 2015

1. Vypočítejte: 5 b.

1 b. a) sin 60° = $\frac{\sqrt{3}}{2}$

1 b. b) sin² 60° = $(\sin 60°)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

1 b. c) sin (60°)² = $\sin 3600° = \sin 0° = 0$

1 b. d) sin² (60°)² = 0

1 b. e) 4 . sin² (60°)² = 0

2. Řešte v R rovnice:

$1+1+2+2+2+3 = 11$ b.

1 b. a) sin x = 0,5 $\quad K_R = \left\{\frac{\pi}{6} + 2k\pi; \frac{5}{6}\pi + 2k\pi\right\}$

1 b. b) sin x = – 0,5 $\quad K_R = \left\{\frac{7}{6}\pi + 2k\pi; \frac{11}{6}\pi + 2k\pi\right\}$

2 b. c) sin (2 x – $\frac{\pi}{2}$) = 0,5

1 b. → $2x - \frac{\pi}{2} = \frac{\pi}{6} + 2k\pi \vee 2x - \frac{\pi}{2} = \frac{5}{6}\pi + 2k\pi$

$2x = \frac{2}{3}\pi + 2k\pi \vee 2x = \frac{4}{3}\pi + 2k\pi$

1 b. → $x = \frac{\pi}{3} + k\pi \vee x = \frac{2}{3}\pi + k\pi$

$K_R = \left\{\frac{\pi}{3} + k\pi; \frac{2}{3}\pi + k\pi\right\}$

2 b. d) sin (2 x + $\frac{\pi}{4}$) = – 0,5

1 b. → $2x + \frac{\pi}{4} = \frac{7}{6}\pi + 2k\pi \vee 2x + \frac{\pi}{4} = \frac{11}{6}\pi + 2k\pi$

$2x = \frac{11}{12}\pi + 2k\pi \vee 2x = \frac{19}{12}\pi + 2k\pi$

1 b. → $x = \frac{11}{24}\pi + k\pi \vee x = \frac{19}{24}\pi + k\pi$

$K_R = \left\{\frac{11}{24}\pi + k\pi; \frac{19}{24}\pi + k\pi\right\}$

2 b. e) cotg (x + π) = $\frac{\sqrt{3}}{3}$

1 b. → $x + \pi = \frac{\pi}{3} + k\pi$

1 b. → $x = -\frac{2}{3}\pi + k\pi$

$K_R = \left\{-\frac{2}{3}\pi + k\pi\right\}$

3 b. e) 2 sin² x – 3 cos x = 0

$\sin^2 x = 1 - \cos^2 x$

$2(1 - \cos^2 x) - 3\cos x = 0$

1 b. → $2\cos^2 x + 3\cos x - 2 = 0$

$D = 9 + 16 = 25$

$\sqrt{D} = 5$

$\cos x = \frac{-3 \pm 5}{4} = \begin{cases} \frac{1}{2} \\ -2 \end{cases}$ 1 b.

$\cos x = \frac{1}{2} \vee \cos x = -2$

$K_R = \left\{\frac{\pi}{3} + 2k\pi; \frac{5}{3}\pi + 2k\pi\right\}$ ← 1 b.

5 b.

3. Narýsujte grafy funkcí v intervalu <0; 2π> (grafy rozlište jasně barvou či druhem čáry):

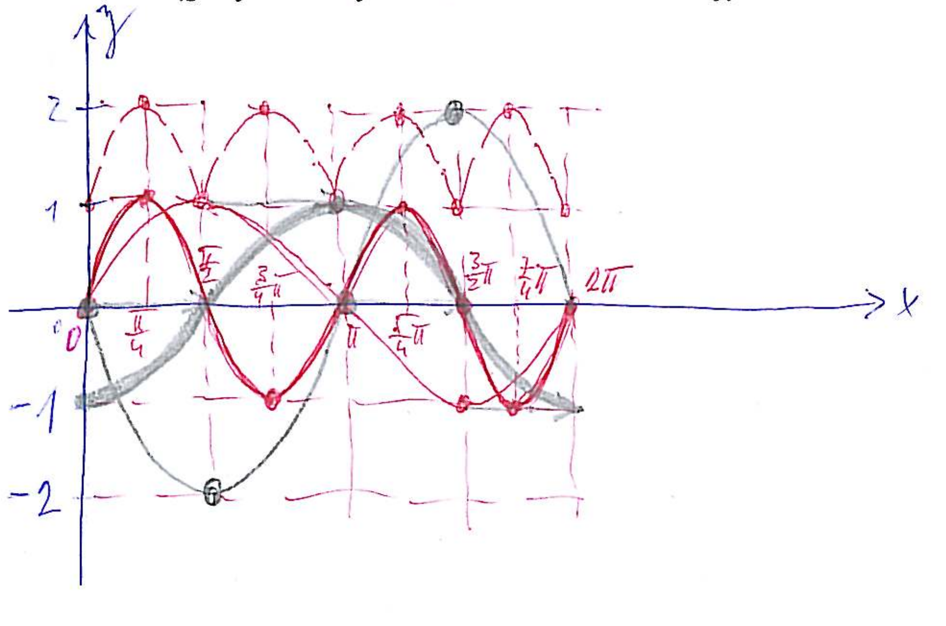
→ a) y = sin x 1 b.

→ b) y = – 2 sin x 1 b.

→ c) y = sin (x – $\frac{\pi}{2}$) 1 b.

→ d) y = sin 2x 1 b.

→ e) y = |sin 2x| + 1 1 b.



(A)

(6b.) 4. Pomocí vhodného vzorce zjednodušte daný výraz:

2b. a) $\sin(\pi + x) = \sin\pi \cos x + \cos\pi \sin x = 0 \cdot \cos x + (-1) \cdot \sin x = \underline{-\sin x}$

2b. b) $\cos(x - \pi) = \cos x \cos\pi + \sin x \sin\pi = \cos x \cdot (-1) + \sin x \cdot 0 = \underline{-\cos x}$

2b. c) $\cos(x + \pi) + \cos(x - \pi) = \cos x \cos\pi - \sin x \sin\pi + \cos x \cos\pi + \sin x \sin\pi =$

1b. ... správné použití vzorců
1b. ... — " — výsledek

$= 2\cos x \cos\pi = 2\cos x(-1) = \underline{-2\cos x}$

nebo

$\cos(x+\pi) + \cos(x-\pi) = 2\cos\frac{(x+\pi)+(x-\pi)}{2} \cdot \cos\frac{(x+\pi)-(x-\pi)}{2} = -2\cos x$

(6b.) 5. Zjednodušte daný výraz, určete definiční obor výrazu:

2b. a) $\sin^4 x - \cos^4 x + \cos^2 x = \underbrace{(\sin^2 x + \cos^2 x)}_{=1}(\sin^2 x - \cos^2 x) + \cos^2 x = \sin^2 x - \cos^2 x + \cos^2 x = \underline{\sin^2 x}$

$D = R$

1b. ... správné zjednodušení
1b. ... — " — určení D

2b. b) $\cos^2 x \cdot \mathrm{tg}^2 x + \cos^2 x = \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x} + \cos^2 x = \sin^2 x + \cos^2 x = \underline{1}$

$D = R - \{\frac{\pi}{2} + k\pi\}$

2b. f) $\frac{\cos^2 x}{1+\sin x} = \frac{1-\sin^2 x}{1+\sin x} = \frac{(1-\sin x)(1+\sin x)}{1+\sin x} = \underline{1-\sin x}$

$1 + \sin x \neq 0$
$\sin x \neq -1$
$x \neq \frac{3}{2}\pi + 2k\pi$

$D = R - \{\frac{3}{2}\pi + 2k\pi\}$

(2b.) 6. Dokažte, že pro všechna x, y ∈ R platí: cos (x + y) + cos (x – y) = 2 · cos x · cos y.

$\cos(x+y) + \cos(x-y) = \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y = 2\cos x \cos y$

nebo

$\cos(x+y) + \cos(x-y) = 2\cos\frac{(x+y)+(x-y)}{2}\cos\frac{(x+y)-(x-y)}{2} = 2\cos x \cos y$

(4b.) 7. Vypočítejte sin x, tg x a cotg x, znáte-li: $\cos x = \frac{3}{5} \wedge x \in (\frac{3}{2}\pi; 2\pi)$.

$x \in (\frac{3}{2}\pi; 2\pi) \Rightarrow \sin x < 0 \wedge \mathrm{tg}\, x < 0 \wedge \mathrm{cotg}\, x < 0$

$\sin^2 x + \cos^2 x = 1 \Rightarrow |\sin x| = \sqrt{1-\cos^2 x} = \sqrt{1-(\frac{3}{5})^2} = \sqrt{\frac{16}{25}} = \frac{4}{5} \Rightarrow$

$\Rightarrow \underline{\sin x = -\frac{4}{5}}$ (2b.)

$\underline{\mathrm{tg}\, x} = \frac{\sin x}{\cos x} = \frac{-\frac{4}{5}}{\frac{3}{5}} = \underline{-\frac{4}{3}}$ 1b.

$\underline{\mathrm{cotg}\, x = -\frac{3}{4}}$ 1b.